# Pion photoproduction and $\gamma N \leftrightarrow \Delta$ amplitudes

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**Abstract**

We review a dynamical model for the pion photoproduction on the nucleon. With the model, we explore sensitivities of observables to the E2 multipole amplitude in the $\gamma N \leftrightarrow \Delta$ transition. It will be demonstrated that the cross section with polarized photons has a significant sensitivity to the E2 amplitude.

## 1. Introduction

Study of the M1 and E2 amplitudes of the $\gamma N \leftrightarrow \Delta$ transition has been done by many authors both experimentally and theoretically. It has been known that the tensor interaction between quarks gives the D-state admixture in the predominant S-state wave functions of the nucleon and the $\Delta$. Non-vanishing E2 amplitude is one of the signals of the D-state admixture. Therefore it is extremely important to determine the size of the E2 amplitude in order to test quark model predictions. However, it is extremely difficult to determine the E2 amplitude accurately. The main reason is that the E2 amplitude is very small compared with the predominant M1 amplitude. Second, a model dependence is unavoidable in separating the background amplitude to extract the resonance amplitude. In this paper, we would like to address two questions. (i) What is model dependent and what is model independent ? (ii) What is the most sensitive observable to the E2 amplitude ? In section 2, we derive the Watson theorem. A dynamical model of Nozawa Blankleider Lee (the NBL model) will be introduced in section 3. Numerical results for the M1 and E2 amplitudes will be presented in section 4. In section 5, the E2/M1 sensitivity will be explored with polarized photon cross sections.

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## 2. The Watson Theorem

Let us first derive the Watson theorem[1]. It requires (i) the unitarity of the S-matrix and (ii) the time-reversal invariance of the T-matrix. The unitarity condition for each partial wave implies

$$S^l S^{l\dagger} = I, \tag{1}$$

where $l \equiv L_{2T2J}$ denotes the partial wave ($P_{33}$, etc.) and $I$ is the unit matrix. The T-matrix is defined by

$$S^l = I - 2\pi i \rho T^l, \tag{2}$$

where $\rho$ is the phase space factor, and $S^l$ and $T^l$ are

$$S^l = \begin{pmatrix} s^l_{\pi\pi} & s^l_{\pi\gamma} \\ s^l_{\gamma\pi} & s^l_{\gamma\gamma} \end{pmatrix}, \quad T^l = \begin{pmatrix} t^l_{\pi\pi} & t^l_{\pi\gamma} \\ t^l_{\gamma\pi} & t^l_{\gamma\gamma} \end{pmatrix}. \tag{3}$$

Here $\pi\pi$, $\pi\gamma$, $\gamma\pi$ and $\gamma\gamma$ denote $\pi N \to \pi N$, $\pi N \to \gamma N$, $\gamma N \to \pi N$ and $\gamma N \to \gamma N$, respectively. Inserting eqs. (2) and (3) into eq. (1), one obtains four coupled equations. The relevant piece for the photoproduction is

$$t^l_{\gamma\pi} - t^{l*}_{\pi\gamma} = -2\pi i \rho (t^l_{\pi\pi} t^{l*}_{\gamma\pi} + t^l_{\pi\gamma} t^{l*}_{\gamma\gamma}). \tag{4}$$

Assuming the time-reversal invariance of the T-matrix, i.e. $t^l_{\pi\gamma} = t^l_{\gamma\pi}$ and dropping the second term of RHS which is suppressed by a factor $\alpha\left(=\frac{1}{137}\right)$, eq. (4) is simplified.

$$t^l_{\gamma\pi} \simeq (1 - 2\pi i \rho t^l_{\pi\pi}) t^{l*}_{\gamma\pi} = e^{2i\delta^l_{\pi\pi}}\, t^{l*}_{\gamma\pi}. \tag{5}$$

Multiplying $t^l_{\gamma\pi}$ to eq. (5), one finally obtains the statement of the Watson theorem.

$$t^l_{\gamma\pi} = |t^l_{\gamma\pi}| e^{i\delta^l_{\pi\pi}}. \tag{6}$$

Namely, the pion photoproduction amplitude has the same phase $e^{i\delta^l_{\pi\pi}}$ as the $\pi N$ scattering. It is important to note that the Watson theorem is model independent.

Let us now consider a case that the amplitude contains resonance (R) and background (B) components, for example, $P_{33}$. The T-matrices are decomposed into

$$t_{\pi\pi} = t^R_{\pi\pi} + t^B_{\pi\pi} \tag{7.a}$$

$$t_{\gamma\pi} = t^R_{\gamma\pi} + t^B_{\gamma\pi}. \tag{7.b}$$

Note that superscript $l$ has been dropped in eq. (7). Inserting eq. (7) into eq. (4), one finds that the background amplitude is unitary, whereas the resonance amplitude is not. The background amplitude is expressed by

$$t^B_{\gamma\pi} = |t^B_{\gamma\pi}| e^{i\delta_B}, \tag{8}$$

where $\delta_B$ is the background $\pi N$ phase shift. Now, question is how to unitarize RHS of eq.

(7.b). In fact, the unitarization method is not unique. For example, Olsson[2] introduced the following method. (i) First assume that the resonance amplitude is modified by a multiplicative phase factor $e^{i\phi}$, i.e.

$$t_{\gamma\pi}^{R} \to |t_{\gamma\pi}^{\Delta}| e^{i(\delta_{P33}+\phi)}, \tag{9}$$

where $t_{\gamma\pi}^{\Delta}$ is the unitary $\Delta$-resonance amplitude. (ii) Then impose the Watson theorem to determine $\phi$. This implies the following condition.

$$|t_{\gamma\pi}| e^{i\delta_{P33}} = |t_{\gamma\pi}^{\Delta}| e^{i(\delta_{P33}+\phi)} + |t_{\gamma\pi}^{B}| e^{i\delta_{B}}, \tag{10}$$

where $\delta_{P33}$ is the $P_{33}$ $\pi N$ phase shift. The parameter $\phi$ is determined as follows.

$$\sin\phi = \frac{|t_{\gamma\pi}^{B}|}{|t_{\gamma\pi}^{\Delta}|} \sin(\delta_{P33} - \delta_{B}) \tag{11.a}$$

$$|t_{\gamma\pi}| = |t_{\gamma\pi}^{\Delta}| \frac{\sin(\delta_{P33} + \phi - \delta_{B})}{\sin(\delta_{P33} - \delta_{B})}. \tag{11.b}$$

Note that $|t_{\gamma\pi}^{\Delta}|$ in eq. (11) is in general $|t_{\gamma\pi}^{R}|$ as shown in Refs. 3 and 4. It is important to note that eq. (11) has been derived with the assumption of eq. (9). We will compare this unitarization method with the coupled channel approach in the next section.

## 3. The NBL Model

We will briefly describe a dynamical model of Nozawa, Blankleider and Lee[5]. There exists other dynamical models by Tanabe and Ohta[4] and by Yang[6] which were constructed in the same spirit. The model starts with the coupled channel Lippmann-Schwinger equation.

$$T = V + TG_0V, \tag{12}$$

where $G_0$ is a free $\pi N$ propagator. The potential $V$ is given by

$$V = \begin{pmatrix} v_{\pi\pi} & v_{\pi\gamma} \\ v_{\gamma\pi} & v_{\gamma\gamma} \end{pmatrix}. \tag{13}$$

Inserting eqs. (3) and (13) into eq. (12), one obtains the following equations.

$$t_{\pi\pi} = v_{\pi\pi} + t_{\pi\pi} G_0 v_{\pi\pi}. \tag{14.a}$$

$$t_{\gamma\pi} = v_{\gamma\pi} + t_{\pi\pi} G_0 v_{\gamma\pi}. \tag{14.b}$$

$$t_{\gamma\gamma} = v_{\gamma\gamma} + t_{\gamma\pi} G_0 v_{\gamma\pi}. \tag{14.c}$$

In deriving eq. (14), we have dropped terms suppressed by a factor $\alpha$. Solving the integral equation of eq. (14.a) for a given $v_{\pi\pi}$, one obtains $t_{\pi\pi}$. Inserting this into eq. (14.b) and integrating over intermediate $\pi N$ states, the pion photoproduction $t_{\gamma\pi}$ matrix is obtained. Similarly, the Compton scattering T-matrix is derived by eq. (14.c).

Let us now consider the $P_{33}$ partial wave. The amplitude is decomposed into resonance and background components as shown in eq. (7). The background $t^{B}_{\gamma\pi}$ matrix satisfies

$$t^{B}_{\gamma\pi} = v^{B}_{\gamma\pi} + t^{B}_{\pi\pi} G_0 v^{B}_{\gamma\pi}. \tag{15}$$

It is therefore separately unitary (see eq. (8)). Furthermore the resonance amplitude $t^{R}_{\gamma\pi}$ has two components.

$$t^{R}_{\gamma\pi} = t^{\Delta}_{\gamma\pi} + t^{VR}_{\gamma\pi}. \tag{16}$$

The first term $t^{\Delta}_{\gamma N}$ is the unitary $\Delta$-amplitude, i.e.

$$t^{\Delta}_{\gamma\pi} = |t^{\Delta}_{\gamma\pi}| e^{i\delta_{P33}}. \tag{17}$$

It should be emphasized that that the $\gamma N\Delta$-vertex has the bare coupling constants $G_M$ and $G_E$, whereas the the $\pi N\Delta$-vertex and the $\Delta$-propagator are all dressed. The second term $t^{VR}_{\gamma\pi}$ is the rescattering amplitude which gives dressing of the $\gamma N\Delta$-vertex. We call it the vertex renormalization (VR) amplitude. Equation (7.b) now becomes

$$t_{\gamma\pi} = t^{\Delta}_{\gamma\pi} + t^{VR}_{\gamma\pi} + t^{B}_{\gamma\pi}. \tag{18}$$

It is important to note that eq. (18) is a general consequence of the present approach based on the coupled channel Lippmann-Schwinger equation. Comparing eq. (18) with RHS of eq. (10), it is clear that $t^{VR}_{\gamma\pi}$ is the dynamical origin of the parameter $\phi$ introduced in Olsson's unitarization method. It should be noted that the additive $t^{VR}_{\gamma\pi}$ amplitude modifies the $\Delta$-amplitude, whereas a multiplicative phase $e^{i\phi}$ does in eq. (10). In the coupled channel approach, the unitarity is guaranteed by the $t^{VR}_{\gamma\pi}$ term. The parameter $\phi$ is no longer necessary. However, this approach requires the knowledge of the half-off-shell $t_{\pi\pi}$ matrix, where the model dependence does come in.

The construction of the NBL model is as follows. (i) The model assumes separable forms for the $\pi N$ potential $v_{\pi\pi}$. This has an advantage that the integral equation (14.a) can be solved analytically and therefore the $\pi N$ T-matrix $t_{\pi\pi}$ has the analytic form. For $P_{11}$ and $P_{33}$ partial waves, the potential consists from resonance and background terms, whereas other partial waves are parameterized in terms of 2-term separable potentials. All parameters in thepotential are fix by fitting $\pi N$ phase shift data up to the pion kinetic energy $E_{lab} = 500$ MeV. (ii) The pion photoproduction potential $v_{\gamma\pi}$ is the Born amplitude with the pseudovector $\pi N$ Lagrangian plus $\rho$- and $\omega$-exchange diagrams. It should be noted that the model satisfies the gauge invariance. The $\gamma N\Delta$-vertex has two coupling constants for the real photon case, i.e. $G_M$ and $G_E$. They are called the magnetic dipole

(M1) and the electric quadrupole (E2) coupling constants, respectively. In the NBL model, they are treated as free parameters. The model has the third parameter $\Lambda$ by introducing a cut-off form factor

$$F_{\mathrm{cut}}(k) = \frac{k^2}{k^2 + \Lambda^2} \tag{19}$$

in eq. (14.b) in order to make the integral over the momentum $k$ converge.

## 4. M1 and E2 Amplitudes

The three parameters $G_M$, $G_E$ and $\Lambda$ are determined by the following manner. For a given $\Lambda$, we determine $G_M$ and $G_E$ to give a best fit to the M1 and E2 amplitudes. We obtained the following results. (i) For $\Lambda$=350 MeV/c, $G_M$=2.80 and $G_E$=0.05. (ii) For $\Lambda$=650 MeV/c, $G_M$=2.28 and $G_E$=0.07. (iii) For $\Lambda$=900 MeV/c, $G_M$=2.30 and $G_E$=0.08. The ratios of the E2 and M1 amplitudes correspond to (i) E2/M1=−1.8%, (ii) E2/M1=−3.1% and (iii) E2/M1=−3.5%, respectively. These three cases give equally good fit to the M1 and E2 amplitudes. However, the case (ii) was found to give an over-all best agreement for differential cross section data. In Fig. 1, we display the result of the M1 and E2 multipoles for the case (ii) E2/M1=−3.1%.

The solid curve is the full amplitude $t_{\gamma\pi}$. The dashed curve, dot-dashed curve and dot-dot-dashed curve are extractions of the $t^B_{\gamma\pi}$, $t^{\Delta}_{\gamma\pi}$ and $t^B_{\gamma\pi}+t^{VR}_{\gamma\pi}$ amplitudes, respectively. The circle, triangle and square correspond to the result of the multipole analyses by

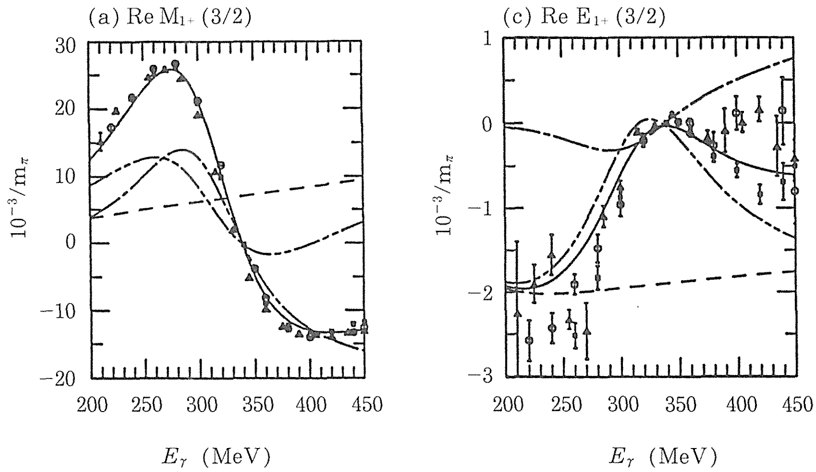


**Fig. 1** M1 and E2 multipole amplitudes

Refs. 7, 8 and 9, respectively. The background amplitude $t_{\gamma\pi}^{B}$ has a smooth energy dependence as expected. The values of $t_{\gamma\pi}^{B}$ agree with the result of Refs. 12 and 13 in Ref. 10. The background amplitude is significantly large for the E2 amplitude. The resonance-like energy dependence of the dot-dot-dashed curve is due to the vertex renormalization amplitude $t_{\gamma\pi}^{VR}$ .

Let us now compare the obtained result E2/M1=−3.1%(5) with the literature. The values are E2/M1=−(0.59±1.01)% to −(2.25±1.02)%(11), −(1.5±0.72)%(10), −4%(6), 0%(12) and +4%(4). The following comment should be noted. The K-matrix formalism was used in Ref. 11. Although the K-matrix $K_{\pi\pi}$ contains a background contribution, the resulted T-matrix $t_{\pi\pi}$ contains no background $t_{\pi\pi}^{B}$ . According to these E2/M1 values, it is clear that there is a significant model dependence the extraction. This might be due to the following reasons. (i) Different unitarization methods used. As mentioned earlier, Refs. 5 and 10 gave a similar background $t_{\gamma\pi}^{B}$ contribution. Therefore the difference must come from $t_{\gamma\pi}^{R}$, namely due to different unitarization methods. Olsson's method and its variations were used in Refs. 3 and 10−12, whereas the coupled channel method with dynamical models was used in Refs. 4−6. It is also evident that there is a significant model dependence among the dynamical models(4)(5)(6). (ii) This will be probably due to different half-off-shell $\pi N$ T-matrices. As far as the present situation is concerned, all we can say about the E2/M1 ratio is that it is small, a few percent with probably a negative sign.

## 5. Sensitivity of the E2 Amplitude

Various predictions of the NBL model for differential cross sections and asymmetries have been given in Ref. 5. In this paper, special attention will be payed to the differential cross sections for unpolarized photons ($\sigma_{\mathrm{unpol}}$), for photons polarized parallel to the production plane ($\sigma_{\parallel}$) and for photons polarized perpendicular to the production plane ($\sigma_{\perp}$)(13). Here $\sigma_{\mathrm{unpol}}$ is the average of $\sigma_{\parallel}$ and $\sigma_{\perp}$. Note that the cross sections $\sigma_{\mathrm{unpol}}$, $\sigma_{\parallel}$ and $\sigma_{\perp}$ become identical at $\theta$=0 and $\pi$, where they are equally sensitive to the E2 amplitude. However, it is difficult to detect pions at the forward and backward angles and no data are presently available there. We therefore study the cross sections near $\theta=\frac{\pi}{2}$, which is preferred experimentally.

Keeping S, P and D-wave multipoles, one can write the cross sections at $\theta=\frac{\pi}{2}$ as

$$\frac{d\sigma_{\perp}\left(\frac{\pi}{2}\right)}{d\Omega}=\frac{k}{\omega_q}\{|E_{0+}-D_{\perp}|^2+|P_{\perp}|^2\} \tag{20.a}$$

$$\frac{d\sigma_{\parallel}\left(\frac{\pi}{2}\right)}{d\Omega}=\frac{k}{\omega_q}\{|E_{0+}-D_{\parallel}|^2+|P_{\parallel}|^2\}, \tag{20.b}$$

where $k$ and $\omega_q$ are the pion momentum and the photon energy in the CM system. In eq. (20), $E_{0+}$ is the S-wave amplitude, and $P_{\perp}$ and $P_{\parallel}$ are P-wave amplitudes given by $P_{\perp}=2M_{1+}+M_{1-}$ and $P_{\parallel}=3E_{1+}-M_{1+}+M_{1-}$. Similarly, $D_{\perp}$ and $D_{\parallel}$ are D-wave amplitudes. It is evident that at $\theta=\frac{\pi}{2}$, $\sigma_{\parallel}$ has a maximum sensitivity to E2, whereas $\sigma_{\perp}$ has no sensitivity. We define $R_{\alpha}$ by the ratio of the cross sections with and without the resonance E2 amplitude. Here $\alpha$ denotes unpol, $\perp$ and $\parallel$. The numerical results for $R_{\alpha}$ are shown in Fig. 2. For E2/M1=−3.1%, $R_{\parallel}$ is increased by 15% at $\theta=\frac{\pi}{2}$, whereas $R_{\text{unpol}}$ and $R_{\perp}$ have much smaller effects. The measurement of $\sigma_{\parallel}$ will be therefore a sensitive observable of the resonance E2 amplitude.

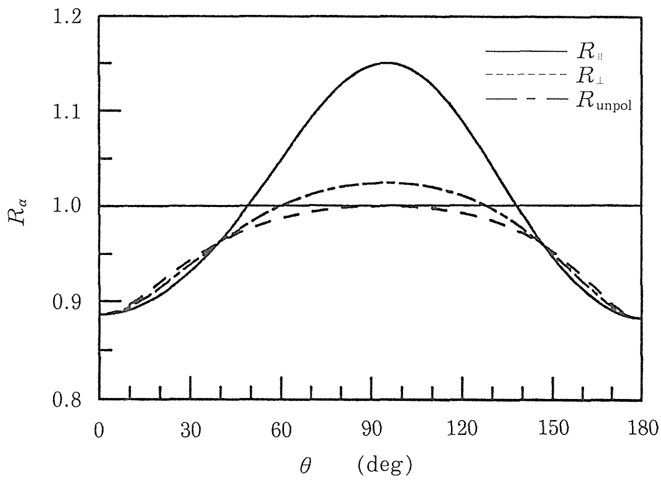


**Fig. 2** Calculated ratios $R_{\parallel}$, $R_{\perp}$ and $R_{\text{unpol}}$ at $E_{\gamma}$=350 MeV

In summary, we have reviewed the coupled channel method with a dynamical model of the pion photoproduction. A detailed comparison has been made between the coupled channel approach and Olsson's unitarization method. A sensitivity study has been also made for the E2 amplitude using cross sections with polarized photons.


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